Cosmological Relativity: Determining the Universe by the Cosmological Redshift

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Abstract

Using cosmological relativity theory, we derive the formula for the cosmological redshift written explicitly in terms of $(1-\Omega)$, where $\Omega = \rho/\rho_c$ is the ratio of the average mass density to the critical "closure" density. Based on the present-day data of observed redshifts, we conclude that $\Omega < 1$.

1 Introduction

In spite of the advances made in recent years in cosmology, the question of what kind of universe we live in is still unsettled [1-10]. According to FRW, based on general relativity theory, one can calculate a critical mass density ρ_c at the nowadays time such that the Hubble expansion will ultimately be reversed if, and only if, the actual average mass density ρ exceeds ρ_c, where $\rho_c = 3H_0^2/8\pi G$, with H_0 the present-time Hubble parameter and G the Newtonian gravitational constant. The value of ρ_c is about 10^{-29}g/cm^3, a few hydrogen atoms per cubic meter throughout the cosmos. Thus it is the value of $\Omega = \rho/\rho_c$ which determines the behavior of the expansion of the universe: $\Omega > 1$, a finite universe, $\Omega < 1$, an infinite curved universe, $\Omega = 1$ an infinite flat-space universe, and the sign of the quantity $(1 - \Omega)$ is the determining factor here [11,12].

In this paper we use cosmological general relativity theory [13-15] to derive a general formula for the redshift in which the term $(1 - \Omega)$ appears explicitly. Since there are enough data of measurements of redshifts, this allows one to determine what is the sign of $(1-\Omega)$, positive, zero or negative. Our conclusion is that $(1 - \Omega)$ cannot be negative or zero. This means that the universe is infinite, curved and expands forever, a result favored by some cosmologists [16]. To this end we proceed as follows.

2 Gravitational Field Equations

We seek a spherical symmetric solution to the Einstein field equations $G_\mu^\nu = \kappa T_\mu^\nu$ and use spherical coordinates $x^\mu = (x^0, x^1, x^2, x^3) = (\tau v, r, \theta, \phi)$, where τ is Hubble's time in the zero-gravity limit and v is the velocity parameter. Since the universe is spherically symmetric at any chosen point, the line element we seek is of the form

$$ds^2 = \tau^2 dv^2 - e^\lambda dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right), \tag{1}$$

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where comoving coordinates, as in the Friedmann theory, are used and λ is a function of the radial distance r only. To determine λ it is enough to solve the field equation $G_0^0 = \kappa T_0^0$ which turns out to be [13-15]

$$G_0^0 = e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = \frac{8\pi G}{c^4} T_0^0 = \frac{8\pi G}{c^2} \rho_{eff}, \tag{2}$$

where a prime denotes derivation with respect to r and $\rho_{eff} = \rho - \rho_c$. The solution of Eq. (2) is

$$e^{-\lambda} = 1 + \frac{(1-\Omega)}{c^2\tau^2} r^2, \tag{3}$$

with $g_{11} = -e^{\lambda}$, $a^2 = c^2\tau^2/(1-\Omega)$ and $\Omega = \rho/\rho_c$.

3   Cosmological Redshift

Having the metric tensor we may now find the redshift of light emitted in the cosmos. As usual, at two points 1 and 2 we have for the wave lengths:

$$\frac{\lambda_2}{\lambda_1} = \frac{ds\,(1)}{ds\,(2)} = \sqrt{\frac{g_{11}\,(1)}{g_{11}\,(2)}}. \tag{4}$$

Using now the solution for g_{11} in Eq. (4) we obtain

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{1 + r_2^2/a^2}{1 + r_1^2/a^2}}. \tag{5}$$

For a sun-like body located at the coordinates origin, and an observer at a distance r from the center of the body, we then have $r_2 = r$ and $r_1 = 0$, thus

$$\frac{\lambda_2}{\lambda_1} = \sqrt{1 + \frac{r^2}{a^2}} = \sqrt{1 + \frac{(1-\Omega)\,r^2}{c^2\tau^2}} \tag{6}$$

for the cosmological contribution to the redshift. If, furthermore, $r \ll a$ we then have

$$\frac{\lambda_2}{\lambda_1} = 1 + \frac{r^2}{2a^2} = 1 + \frac{(1-\Omega)\,r^2}{2c^2\tau^2}, \tag{7}$$

to the lowest apprixomation in r^2/a^2, and thus

$$z = \frac{\lambda_2}{\lambda_1} - 1 = \frac{r^2}{2a^2} = \frac{(1-\Omega)\,r^2}{2c^2\tau^2}. \tag{8}$$

From Eqs. (6)–(8) it is clear that Ω cannot be larger than one since otherwise z will be negative, which means blueshift, and as is well known nobody sees such a thing. If $\Omega = 1$ then $z = 0$, and for $\Omega < 1$ we have $z > 0$. The case of $\Omega = 1$ is also implausible since the light from stars we see is usually redshifted more than the redshift due to the gravity of the body emitting the radiation, as is evident from our sun, for example, whose emitted light is shifted by only $z = 2.12 \times 10^{-16}$ [17].

4 Conclusions

One can thus conclude that the theory of cosmological general relativity predicts that the universe is infinite and expands from now on forever. As is well known the standard FRW model does not relate the cosmological redshift to the kind of the universe. Our conclusion is also in full agreement with the measurements recently obtained by the *High-Z Supernovae Team* and the *Suprenovae Cosmology Project* [18-24].

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